# Spin alignment of the high $p_T$ vector mesons in polarized $pp$ collisions at high energies

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Using the same method as that used in $e^+e^-$ annihilation and lepton-nucleon deep inelastic scattering, we calculate the spin alignment of vector mesons with high transverse momenta in high energy $pp$ collisions with one longitudinally polarized beam. We present the results obtained at BNL RHIC energies. We also study the spin alignment of such vector mesons in the case of a transversely polarized proton beam and present its relation to the transversity distribution in the nucleon. Numerical results obtained using simple models for the transversity distributions are also given. These results show that significant spin alignment exists in both cases and can be measured in experiments such as those at the RHIC.

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## I. INTRODUCTION

Spin effects in high energy fragmentation processes have attracted much attention recently (see, e.g., Refs. [1–12] and the references given therein). One of the important issues is the study of spin transfer from the fragmenting quark to the produced hadrons. It can be studied by measuring the polarizations of the hadrons produced in the fragmentation of a polarized quark. Hyperon polarization has frequently been used in this connection [2–10]. On the other hand, it has been pointed out [11,12] that the vector meson polarization can also be used for this purpose since the polarization of a vector meson can also be studied easily in experiments. Furthermore, compared with the hyperon, the production rate of a vector meson is usually higher and the origin is simpler because of the much smaller contribution from the decay of heavier hadrons. This implies fewer theoretical uncertainties in calculations and better statistics in experiments. Therefore, study of the polarization of vector mesons may provide us with important information for understanding the role of spin in hadronic reactions.

The polarization of a vector meson is described by the spin density matrix $\rho$ or its element $\rho_{m,m'}$, where $m$ and $m'$ label the spin component along the quantization axis, usually the $z$ axis of the frame. The diagonal elements $\rho_{11}$, $\rho_{00}$, and $\rho_{-1-1}$ for the unit-trace matrix are the relative intensities of the meson spin component $m$ taking the values $1$, $0$, and $-1$, respectively. In experiments, $\rho_{00}$ can be measured from the angular distribution of its decay products and a deviation of $\rho_{00}$ from 1/3 indicates spin alignment. In the helicity basis, i.e., in the case that the $z$ axis is chosen as the moving direction of the vector meson, the matrix is usually called the helicity density matrix, and $\rho_{00}$ represents the probability of the vector meson to be in the helicity zero state. Measurements have been carried out in different reactions [13–16], in particular, in $e^+e^-$ annihilation at the CERN $e^+e^-$ collider LEP recently [16]. The data [16] show that $\rho_{00}$ is significantly larger than 1/3 for vector mesons with large $z$ and increases with increasing $z$ (here $z \equiv 2p_V/\sqrt{s}$, where $p_V$ is the momentum of the vector meson and $\sqrt{s}$ is the total $e^+e^-$ center of mass energy). This means that significant spin alignments exist for vector mesons in the helicity frame in $e^+e^-$ annihilation at the $Z^0$ pole, and the effect is more significant for the large momentum fraction region. A simple statistical model [17] based on spin counting gives the result $\rho_{00}=1/3$. A QCD-inspired model is given in Ref. [18], where a fast quark combines with a soft antiquark to form a vector meson, and the soft antiquark preferentially has the same helicity as the fast one. This model leads to $\rho_{00}=0$. Another model [19] describes the production of vector mesons through the channel $q \rightarrow qV$, with the vector meson coupling to the quark like a vector current. In this case, the helicity-conserving vector current gives rise to $\rho_{00}=1$. None of these predictions is consistent with the data [16].

In a recent paper [11], we calculated the helicity density matrix of vector mesons inclusively produced in $e^+e^-$ annihilation at the $Z^0$ pole by taking the spin of a vector meson that contains a fragmenting quark as the sum of the spin of the polarized fragmenting quark and that of the antiquark created in the fragmentation process. By comparing the results obtained with the data [16], we showed that the experimental results for $\rho_{00}$ imply a significant polarization for the antiquark that is created in the fragmentation process and combines with the fragmenting quark to form the vector meson. It should be polarized in the opposite direction to that of the fragmenting quark, and the polarization can approximately be written as

$$P_z = -\alpha P_f, \tag{1}$$

where $\alpha \approx 0.5$ is a constant; $P_z$ is the polarization of the antiquark in the direction of movement of the fragmenting quark, and $P_f$ is the longitudinal polarization of the fragmenting quark of flavor $f$. The results for $\rho_{00}$ obtained from the above relation are in good agreement with the data for different mesons. Since the calculations are rather straightforward, the relation given by Eq. (1) can be considered as a direct implication of the data [16] in $e^+e^-$ annihilation. It implies that there exists a spin interaction between the fragmenting quark and the antiquark during the hadronization process. It would be interesting to extend the calculations to other processes and make further tests to see whether the relation is universal, in the sense that it is true for quark fragmentation in all different reactions. Hence we made similar calculations for polarized lepton-nucleon deep inelastic scattering (DIS) and polarized $pp$ collisions. The results for DIS was given in Ref. [12]. In this paper, we present the calculations and results for high $p_T$ vector mesons in polarized $pp$ collisions, where the factorization theorem can be used so that fragmentation effects and other effects such as those from the structure functions can be studied separately.

Compared with those in lepton-induced reactions such as $e^+e^-$ annihilations and DIS, the study of high $p_T$ vector mesons in $pp$ collisions has the following properties. (1) The hard scatterings involved are strong interaction processes rather than the electroweak processes in the lepton-induced reactions, so the corresponding cross section should in general be larger. Furthermore, the luminosity of the incoming proton beams can in general be made higher than that for leptons. Hence, the statistics in experiments can be better. (2) Not only longitudinally but also transversely polarized quarks can be produced, so that we can study the properties in both cases. (3) There are many different hard subprocesses which contribute to the high $p_T$ hadron production. This makes the study more interesting and, at the same time, more complicated than in lepton-induced reactions. For example, gluon fragmentation is also involved here, which is unclear even for the unpolarized case. It is very important to know whether its contribution is significant in different kinematic regions. We first make an estimation of it in the next section using a Monte Carlo event generator and then present the calculation method for the spin alignment of vector mesons in $pp$ collisions. In Sec. III, we calculate the spin alignment of vector mesons with high $p_T$ in the helicity frame in $pp$ collisions with one longitudinally polarized proton beam and present the results. In Sec. IV, we extend the study to the the spin alignment of high $p_T$ vector mesons for the case that one proton beam is transversely polarized. Finally, a short summary is given in Sec. V.

## II. THE CALCULATION METHOD

In this section, we first summarize the calculation method for the spin alignment of vector mesons that was used in $e^+e^-$ annihilation, and then extend it to inclusive high $p_T$ vector meson production in longitudinally polarized $pp$ collisions.

### A. $e^+e^-$ annihilation process

The calculation method for vector mesons produced in $e^+e^-$ annihilation was presented in Ref. [11]. The main points are summarized as follows. For $e^+e^-$ annihilation at a given energy, we need to consider vector mesons produced in the fragmentation of a polarized initial quark $q_f^0$ of flavor $f$ and from a polarized initial antiquark $\overline{q_f^0}$. The polarization of $q_f^0$ or $\overline{q_f^0}$ is a given constant which can be calculated using the standard model for electroweak interaction. To calculate the spin density matrices of these vector mesons, we divide them into the following two groups and consider them separately: (A) those that contain the fragmenting quark $q_f^0$; (B) those that do not contain the fragmenting quark. The spin density matrix $\rho^V(z)$ for the vector meson $V$ is given by

$$\rho^V(z)=\sum_f \frac{\langle n(z|\mathrm{A},f)\rangle}{\langle n(z)\rangle}\rho^V(\mathrm{A},f)+\frac{\langle n(z|\mathrm{B})\rangle}{\langle n(z)\rangle}\rho^V(\mathrm{B}), \quad (2)$$

where $\langle n(z|\mathrm{A},f)\rangle$ and $\rho^V(\mathrm{A},f)$ are, respectively, the average number and spin density matrix of the vector mesons from group A; $\langle n(z|\mathrm{B})\rangle$ and $\rho^V(\mathrm{B})$ are those from group B; $\langle n(z)\rangle=\Sigma_f\langle n(z|\mathrm{A},f)\rangle+\langle n(z|\mathrm{B})\rangle$ is the total number of vector mesons; and $z$ is the momentum fraction of the initial quark's momentum carried off by the vector meson. The average numbers $\langle n(z|\mathrm{A},f)\rangle$ and $\langle n(z|\mathrm{B})\rangle$ are independent of the spin properties and can be calculated using an event generator based on a fragmentation model which gives a good description of the unpolarized data. We used the generator PYTHIA [20] in our calculations.

In terms of the cross section and fragmentation functions usually used, Eq. (2) has the following form:

$$\rho^V(z)=\frac{\sum_f D^{\mathrm{A}}_{V/f}(z)\rho^V(\mathrm{A},f)\sigma_{e^+e^-\to q_f^0\overline{q_f^0}}}{\sum_f D_{V/f}(z)\sigma_{e^+e^-\to q_f^0\overline{q_f^0}}}+\frac{\sum_f D^{\mathrm{B}}_{V/f}(z)\rho^V(\mathrm{B})\sigma_{e^+e^-\to q_f^0\overline{q_f^0}}}{\sum_f D_{V/f}(z)\sigma_{e^+e^-\to q_f^0\overline{q_f^0}}}, \quad (3)$$

where $D^A_{V/f}(z)$ and $D^B_{V/f}(z)$ are the fragmentation functions of the initial quark $q_f^0$ (antiquark) into vector mesons of groups A and B with momentum fraction $z$, respectively; $D_{V/f}(z)=D^A_{V/f}(z)+D^B_{V/f}(z)$ is the usual fragmentation function; $\sigma_{e^+e^-\to q_f^0\overline{q_f^0}}$ is the cross section of the process $e^+e^-\to q_f^0\overline{q_f^0}$. We see that $D^A_{V/f}(z)\sigma_{e^+e^-\to q_f^0\overline{q_f^0}}$ is just the cross section of the production of the meson $V$ which contains the initial quark via the fragmentation of the $f$-flavor quark in $e^+e^-$ annihilation, i.e., $e^+e^-\to q_f^0\overline{q_f^0}\to V(q_f^0\overline{q})X$. It is proportional to the average number $\langle n(z|\mathrm{A},f)\rangle$ in Eq. (2). The denominator is the total cross section of inclusive vector meson production, which corresponds to $\langle n(z)\rangle$ of Eq. (2).

There are many different possibilities for the production of the vector mesons in group B; we take them as unpolarized, i.e., $\rho^V(\mathrm{B})=1/3$. For those vector mesons from group A, i.e., those that contain $q_f^0$ and an antiquark $\overline{q}$ created in the fragmentation, the spin is taken as the sum of the spins of $q_f^0$ and $\overline{q}$. The polarization of $q_f^0$ is taken as the same as that before the fragmentation. Then the spin density matrix $\rho^V(\mathrm{A},f)$ can be calculated from the direct product of the spin density matrix $\rho^{q_f^0}$ for $q_f^0$ and $\rho^{\overline{q}}$ for $\overline{q}$. Transforming the direct product $\rho^{q_f^0\overline{q}}=\rho^{q_f^0}\otimes\rho^{\overline{q}}$ to the coupled basis $|s,s_z\rangle$ (where $\vec{s}=\vec{s}^{\,q}+\vec{s}^{\,\overline{q}}$), we can obtain the spin density matrix of the vector meson $\rho^V(\mathrm{A},f)$. In the helicity frame of $q_f^0$, i.e., the $z$ axis is taken as the direction of movement of $q_f^0$, the density matrix of the vector meson of type A is given by

$$\rho^V(\mathrm{A},f)=\frac{1}{3+P_fP_z}\begin{pmatrix} (1+P_f)(1+P_z) & \frac{1+P_f}{\sqrt{2}}(P_x-iP_y) & 0 \\ \frac{(1+P_f)}{2}(P_x+iP_y) & (1-P_fP_z) & \frac{1-P_f}{\sqrt{2}}(P_x-iP_y) \\ 0 & \frac{1-P_f}{\sqrt{2}}(P_x+iP_y) & (1-P_f)(1-P_z) \end{pmatrix}, \quad (4)$$

where $P_f$ is the longitudinal polarization of $q_f^0$ and $\vec{P}=(P_x,P_y,P_z)$ is the polarization vector of the antiquark $\overline{q}$. Hence, the 00-component of the density matrix takes the following simple form:

$$\rho_{00}^V(\mathrm{A},f)=(1-P_fP_z)/(3+P_fP_z), \quad (5)$$

in which there is only one unknown variable $P_z$, the polarization of $\overline{q}$ along the moving or polarization direction of $q_f^0$.

Using Eqs. (2) and (5), we can determine $P_z$ in different cases by fitting the data [16] in $e^+e^-$ annihilation at the $Z^0$ pole for the production of different vector mesons. As mentioned in the Introduction, we found [11] that the data for different vector mesons can be fitted reasonably if we take $P_z$ satisfying the relation given by Eq. (1). Now we insert Eq. (1) into Eq. (5) and obtain that [21]

$$\rho_{00}^V(\mathrm{A},f)=(1+\alpha P_f^2)/(3-\alpha P_f^2). \quad (6)$$

Finally, from Eqs. (2) and (6), we have

$$\rho_{00}^V(z)=\sum_f \frac{1+\alpha P_f^2}{3-\alpha P_f^2}\frac{\langle n(z|\mathrm{A},f)\rangle}{\langle n(z)\rangle}+\frac{1}{3}\frac{\langle n(z|\mathrm{B})\rangle}{\langle n(z)\rangle}. \quad (7)$$

### B. Polarization of the outgoing quark in the hard subprocess in $pp$ collisions

Now we come to the high $p_T$ vector meson production in polarized $pp$ collisions. Such vector mesons come mainly from the fragmentation of outgoing partons in hard scattering subprocesses. The outgoing parton can be a gluon or a quark, and the cross section of the gluon-involved subprocess is even larger than others. However, the gluon distribution and fragmentation functions are both poorly known yet, especially for the polarized case. Fortunately, the momentum fractions carried by the gluons in a proton are usually very small, so the gluon contributions to very high $p_T$ hard scattering subprocesses are suppressed. In addition, it is known that the gluon fragmentation function is softer than the quark fragmentation function. Consequently, for the final hadron production with high $p_T$, the contribution from gluon fragmentation is much smaller than that from quark fragmentation [10]. To see it numerically, we calculated these contributions using PYTHIA. In Fig. 1, we show the results for inclusive $\rho^0$ production with $p_T>13$ GeV at $\sqrt{s}=500$ GeV in $pp$ collisions. We see that, at this high $p_T$ cutoff, the contribution of gluon fragmentation is indeed much smaller than that of quark fragmentation, especially for the large $\eta$ region ($\eta$ is the pseudorapidity of the vector meson). Thus, as we did in studying hyperon polarization in high $p_T$ jets [10], we neglect the contribution of polarized gluon fragmentation to the spin alignment of vector mesons and consider only the contribution of the polarized quark's fragmentation.

For quark fragmentation, we can use the same method of calculating spin alignment as that used in $e^+e^-$ annihilation. To do it, we need to know the polarization of the outgoing quark in the hard subprocess. This quark is also longitudinally polarized when one proton beam is longitudinally polarized and its polarization can be calculated in perturbative QCD (PQCD). The polarization obtained can be written as

$$P_q(x_a,Q^2,y)=D_L(y)P_q^{in}(x_a,Q^2), \quad (8)$$

where $D_L(y)$ is the longitudinal polarization transfer factor in the scattering from the incoming parton to the outgoing parton; $D_L(y)$ is a function only of $y$ which is defined as $y\equiv k_b\cdot(k_a-k_c)/k_a\cdot k_b$ for the subprocess $ab\to cd$, where $k_a$, $k_b$, $k_c$, and $k_d$ are the four-momenta of the partons $a$, $b$, $c$, and $d$ respectively. $D_L(y)$ has been calculated using PQCD and the leading order results can be found in many publications, for example, in Ref. [10]. We take the parton $a$ as the parton from the polarized proton beam, and the polarization of the incoming parton $a$ is given by $P_q^{in}(x_a,Q^2)$

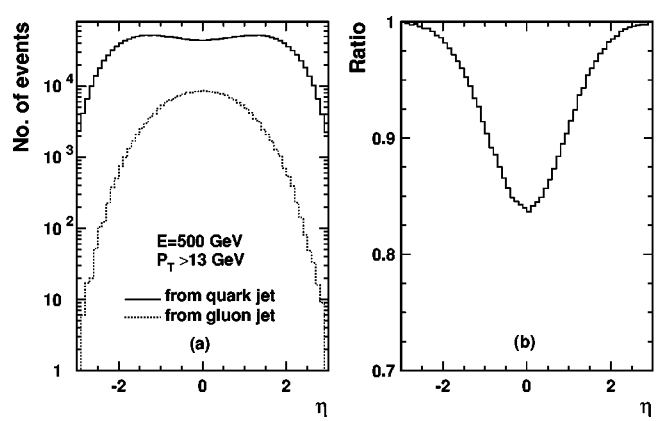


FIG. 1. (a) Contributions of quark and gluon fragmentation to $\rho^0$ production, and (b) the ratio of the quark fragmentation's contribution to the total rate in $pp\to\rho^0X$ at $\sqrt{s}=500$ GeV and $p_T>13$ GeV.

$=\Delta q(x_a,Q^2)/q(x_a,Q^2)$, where $\Delta q(x_a,Q^2)$ and $q(x_a,Q^2)$ are the helicity and unpolarized distribution functions at momentum fraction $x_a$ and scale $Q^2$. Therefore, the longitudinal polarization $P_q$ is a function of the variables $x_a$, $Q^2$, and $y$. Since we are now discussing $pp$ collisions with one polarized beam, it is clear that the outgoing quarks with positive momenta in the $z$ direction have a much larger probability of being polarized than those with negative momenta when the positive axis of $z$ is taken as the direction of the incoming polarized proton.

### C. Spin alignment in longitudinally polarized $pp$ collisions

As we did for $e^+e^-$ annihilation, to calculate the spin density matrix of the high $p_T$ vector mesons in $pp$ collisions, we also divide them into two groups A and B according to whether or not they contain an outgoing quark in the hard scattering and consider them separately. The difference is that now the fragmenting quark is an outgoing quark in the hard scattering subprocesses and its polarization is not a constant. It depends on the origin of this quark and is given by Eq. (8). Obviously, it is a function of $x_a$, $Q^2$, and $y$. Hence, $\rho_{00}$ for vector mesons of group A should also be a function of $x_a$, $Q^2$, and $y$ and it is given by

$$\rho_{00}^V(x_a,Q^2,y|\mathrm{A})=(1+\alpha P_q^2)/(3-\alpha P_q^2). \tag{9}$$

To obtain $\rho_{00}$ for vector mesons at a given pseudorapidity $\eta$, we need to replace the products in Eq. (3) with the corresponding convolutions, i.e.,

$$\rho^V(\eta)=\frac{\sum_f [D_{V/f}^{\mathrm{A}}(z_c)\otimes\rho^V(\mathrm{A},f)+D_{V/f}^{\mathrm{B}}(z_c)\otimes\rho^V(\mathrm{B})]\otimes\sigma_{pp\to q_fX}}{\sum_f D_{V/f}(z_c)\otimes\sigma_{pp\to q_fX}}. \tag{10}$$

To write it more precisely, we have

$$\rho_{00}^V(\eta)=\frac{\int d^2p_T\sum_{abcd} dx_a dx_b f_a(x_a,\mu^2)f_b(x_b,\mu^2)(d\hat{\sigma}/d\eta)[D_{V/c}^{\mathrm{A}}(z_c,\mu^2)\rho_{00}^V(\mathrm{A},c)+D_{V/c}^{\mathrm{B}}(z_c,\mu^2)\rho_{00}^V(\mathrm{B})]}{\int d^2p_T\sum_{abcd}\int dx_a dx_b f_a(x_a,\mu^2)f_b(x_b,\mu^2)(d\hat{\sigma}/d\eta)(ab\to cd)D_{V/c}(z_c,\mu^2)}, \tag{11}$$

where $p_T$ is the transverse momentum of the vector meson; $f_a(x_a,\mu^2)$ and $f_b(x_b,\mu^2)$ are the unpolarized distribution functions of partons $a$ and $b$ in the proton at the scale $\mu$; $x_a$ and $x_b$ are the corresponding momentum fractions carried by $a$ and $b$; $D_{V/c}(z_c,\mu^2)$ is the usual fragmentation function of parton $c$ into vector meson $V$; $D_{V/c}^A(z_c,\mu^2)$ and $D_{V/c}^B(z_c,\mu^2)$ are those of parton $c$ into vector mesons of groups A and B, respectively; $z_c$ is the momentum fraction of parton $c$ carried by the $V$ produced; $d\hat{\sigma}/d\eta$ is the cross section at the parton level. The cross section of the hard subprocess can be calculated using perturbative QCD. The summation in Eq. (11) runs over all possible subprocesses.

The fragmentation functions are independent of the spin properties and can be calculated using an available Monte Carlo event generator. In practice, we generate a $pp$ collision event and then search for the produced vector meson $V$. We then calculate its contribution to $\rho_{00}^V(\eta)$ by tracing back its origin. If the vector meson $V$ belongs to group B, then its contribution to $\rho_{00}^V(\eta)$ is given by $\rho_{00}(\mathrm{B})=1/3$. If it belongs to group A, we then need to further trace back the origin of the fragmenting quark to calculate its polarization $P_q$ using Eq. (8). We insert $P_q$ into Eq. (9) to get the contribution $\rho_{00}(\mathrm{A})$ of such vector mesons to $\rho_{00}^V(\eta)$. After running the program for a sufficiently large number of events, we obtain the final $\rho_{00}^V(\eta)$. In terms of a mathematical formula, the calculation in this procedure is expressed as

$$\rho_{00}^V(\eta)=\left[\sum_{i=1}^{N(\eta|\mathrm{A})}\rho_{00}^i(x_a,Q^2,y|\mathrm{A})+\frac{1}{3}N(\eta|\mathrm{B})\right]\Big/ N(\eta), \tag{12}$$

where $N(\eta|\mathrm{A})$ and $N(\eta|\mathrm{B})$ are the numbers of vector mesons of groups A and B as a function of $\eta$, and $N(\eta)=N(\eta|\mathrm{A})+N(\eta|\mathrm{B})$ is the total number of vector mesons. The results for $\rho_{00}^V(\eta)$ in longitudinally polarized $pp$ collisions are presented in the next section.

## III. RESULTS FOR THE LONGITUDINALLY POLARIZED CASE

Using the method described in the last section, we calculate the $\rho_{00}$'s for different vector mesons in $pp$ collisions with one proton beam longitudinally polarized. We now summarize the results obtained. Using the generator PYTHIA [20], we first calculate the different contributions to vector meson production, i.e., those of groups A and B. As an example, we show the results for $K^{*+}$ at $\sqrt{s}=500$ GeV and $p_T>13$ GeV in Fig. 2. From these results, we can see clearly that the decay contribution is indeed very small. The contribution of those vector mesons containing an outgoing quark in the hard scattering subprocess is very high, even larger

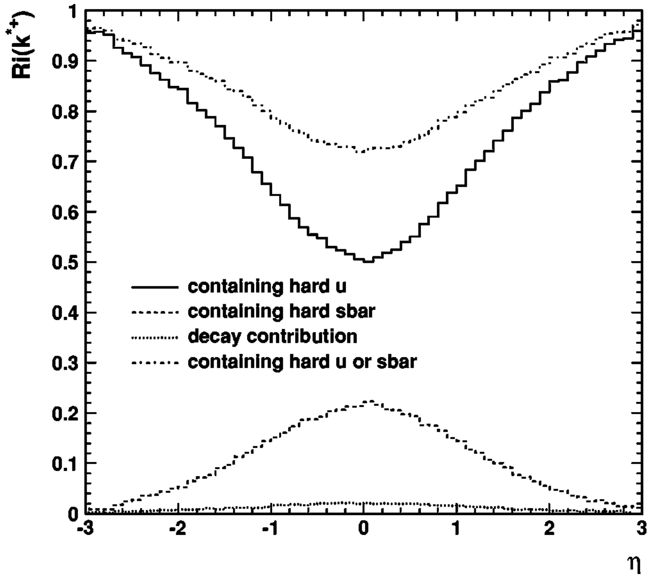


FIG. 2. Different contributions to $K^{*+}$ production in $pp \to K^{*+}X$ at $\sqrt{s}=500$ GeV and $p_T>13$ GeV as functions of $\eta$. Here "containing hard $u$" and similar denote the contributions of those vector mesons containing the outgoing quark from the hard scattering subprocess.

than 90% for the large $\eta$ region. The results for the $\rho_{00}$'s for different vector mesons are shown in Fig. 3. We see that the spin alignments for $K^{*+}$, $\rho^{\pm}$, and $\rho^0$ are significantly high. The $\rho_{00}$'s obtained for these mesons increase from 1/3 to about 0.45 with increasing $\eta$. There is nearly no spin alignment in the negative $\eta$ region since the scattered quark moving in the negative $z$ direction is almost unpolarized. The spin alignment for $K^{*0}$ is smaller in both the positive and negative $\eta$ regions, this is because $d$ quark fragmentation dominates $K^{*0}$ production at high $p_T$ and the polarization of

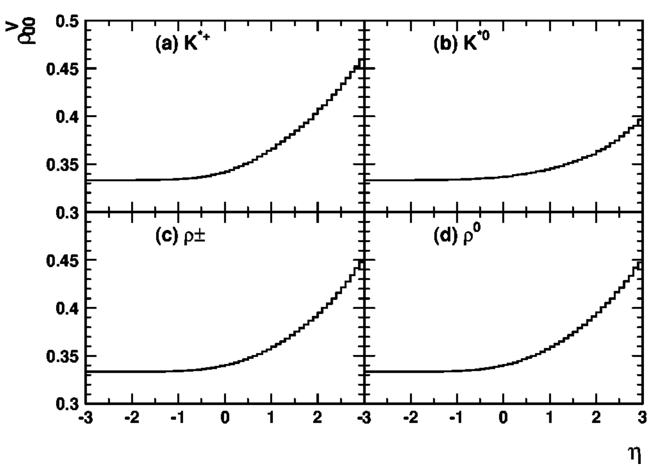


FIG. 3. Spin alignment of different vector mesons with $p_T > 13$ GeV in $pp \to VX$ at $\sqrt{s}=500$ GeV in the helicity frame when one proton beam is longitudinally polarized. The standard LO set of GRSV2000 [27] and LO set of GRV98 [28] are used for the helicity and unpolarized distribution functions respectively. The scale $\mu$ is taken as the transverse momentum of the hard scattering subprocess.

the $d$ quark $|\Delta d(x,Q^2)/d(x,Q^2)|$ is smaller than that of the $u$ quark $|\Delta u(x,Q^2)/u(x,Q^2)|$.

We can also make calculations for other vector mesons, such as $K^{*-}$, $\bar{K}^{*0}$, and $\phi$. However, since these mesons do not contain $u$ or $d$ valence quarks, which dominate the high $p_T$ hadron production in $pp$ collisions, their production rates at high $p_T$ are small. This makes the statistics much worse than those for $K^{*+}$, $K^{*0}$, $\rho^{\pm}$, and $\rho^0$. For the same reason, the $s$ quark distribution is much more important. Hence, the spin alignments for these mesons are much more sensitive to the helicity distribution of the strange quark in the nucleon, which is less precisely determined yet. It is thus not a good choice to study the spin transfer in a fragmentation process by measuring the spin alignment of these mesons. But it is possible to use them to study the helicity distribution of the strange sea in a nucleon if they can be measured with good accuracy.

## IV. RESULTS FOR THE TRANSVERSELY POLARIZED CASE

When one of the proton beams is transversely polarized, the incoming quark $a$ for the elementary hard scattering process can also be transversely polarized in the same direction. Its polarization $P_{aT}$ is determined by the transversity distribution function [22] $\delta q(x_a,Q^2)$ and the unpolarized distribution function $q(x_a,Q^2)$, i.e., $P_{aT} = \delta q(x_a,Q^2)/q(x_a,Q^2)$. There is no gluon transversity distribution at leading twist [22]; thus the incoming gluon in the hard scattering is not transversely polarized, which is different from the longitudinally polarized case. The transverse polarization can also be transferred from the incoming quark to the outgoing quark in the hard scattering. The polarization direction of the outgoing quark is also transverse to the direction of motion of the quark but is in general different from that of the incoming quark. Both the magnitude and the direction of the polarization of the outgoing quark can be calculated [23] using PQCD for the hard elementary process. The results are summarized in the following.

We recall that, for a quark $a$ with transverse polarization $P_{aT}$, the spin density matrix $\rho_a$ in the helicity basis is given by

$$\rho_a^{(in)} = \frac{1}{2}\begin{pmatrix} 1 & P_{aT}e^{-i\phi} \\ P_{aT}e^{i\phi} & 1 \end{pmatrix}, \tag{13}$$

where $\phi$ is the angle between the polarization direction and the $x$ axis. The $xy$ plane is perpendicular to the direction of motion of the quark, which is taken as the $z$ direction; and the $x$ axis is taken as the direction normal to the scattering plane. The spin density matrix of the outgoing quark in the helicity basis can be obtained from $\rho_a^{(in)}$ given in Eq. (13) and the helicity amplitudes of the scattering. The result is given by [23]

$$\rho_q = \frac{1}{2}\begin{pmatrix} 1 & P_{aT}D_T(y)e^{-i\phi} \\ P_{aT}D_T(y)e^{i\phi} & 1 \end{pmatrix}, \tag{14}$$

where $D_T(y)$ is the real function of $y$ defined in Sec. II B for the scattering and is called the polarization transfer factor. $D_T(y)$ values for different subprocesses have been calculated using PQCD and can be found, for example, in Ref. [23].

From Eq. (14), we see that the outgoing quark is also transversely polarized. The magnitude of the polarization is given by $D_T(y)P_{aT}$, i.e.,

$$P_{qT}(x_a,Q^2,y)=D_T(y)\cdot\delta q(x_a,Q^2)/q(x_a,Q^2). \quad (15)$$

We see also that the angle between the polarization direction and the $x$ axis remains the same before and after the scattering. More precisely, the polarization vector of the outgoing quark is in the plane transverse to the direction of motion of the quark, and the angle between the polarization direction and the normal to the scattering plane is the same as that for the incoming quark. In other words, the direction of transverse polarization of the incoming and that of the outgoing quark are related to each other by a rotation around the normal to the scattering plane, which changes the direction of motion of the quark from the incoming to the outgoing direction (cf. Fig. 2 of Ref. [23]). We note that the direction of polarization of the outgoing quark depends not only on the scattering angle but also on the azimuthal direction. In contrast, the magnitude of the polarization depends only on the scattering angle. The dependence is given by the corresponding spin transfer function $D_T(y)$, which is a function only of $y$ defined above.

Having seen that the outgoing quarks are also transversely polarized if the incoming protons are transversely polarized, we now discuss the spin alignment of the vector mesons produced in the fragmentation of such quarks. We recall that the relation given in Eq. (1) is obtained for the fragmentation of longitudinally polarized quarks by fitting the $e^+e^-$ annihilation data [16]. It shows that the antiquark that combines with the fragmenting quark to form the vector meson is polarized in the opposite direction to that of the fragmenting quark. To test whether the relation is true in general for the fragmentation of a longitudinally polarized quark in any high energy reaction, we extended the calculations of the spin alignment of vector mesons to other lepton-induced reactions in [12] and to longitudinally polarized $pp$ collisions in the previous sections. Now we further assume that the relation is also true for the fragmentation of a quark polarized in any direction, i.e., we extend the relation to

$$\vec{P}_{\bar{q}}=-\alpha\vec{P}_q. \quad (16)$$

We use this relation to calculate the spin alignment of the vector mesons in transversely polarized $pp$ collisions so that this extension can be tested in future experiments. The calculations are carried out in a way similar to that in the longitudinally polarized case. We now present the calculations and the results in the following two different frames.

(1) *In the helicity frame*. In the helicity frame of the outgoing quark of the hard scattering, its spin density matrix is off diagonal and is given by Eq. (14). From the relation given by Eq. (16), the spin density matrix of the antiquark that combines with the fragmenting quark to form the vector meson is

$$\rho_{\bar{q}}=\frac{1}{2}\begin{pmatrix} 1 & -\alpha P_{qT}e^{-i\phi} \\ -\alpha P_{qT}e^{i\phi} & 1\end{pmatrix}. \quad (17)$$

We build the direct product of $\rho_q$ given by Eq. (14) with $\rho_{\bar{q}}$ given by Eq. (17), transform it to the coupled basis, and obtain the spin density matrix of the vector meson of group A as

$$\rho^{(h)}(\mathrm{A})=\frac{1}{3-\alpha P_{qT}^2}\begin{pmatrix} 1 & \frac{1}{\sqrt{2}}P_{qT}(1-\alpha)e^{-i\phi} & -\alpha P_{qT}^2e^{-2i\phi} \\ \frac{1}{\sqrt{2}}P_{qT}(1-\alpha)e^{i\phi} & 1-\alpha P_{qT}^2 & \frac{1}{\sqrt{2}}P_{qT}(1-\alpha)e^{-i\phi} \\ -\alpha P_{qT}^2e^{2i\phi} & \frac{1}{\sqrt{2}}P_{qT}(1-\alpha)e^{i\phi} & 1\end{pmatrix}, \quad (18)$$

where the superscript $(h)$ denotes the helicity frame. Hence, $\rho_{00}^{(h)}(\mathrm{A})$ for such vector mesons in the helicity basis is given by

$$\rho_{00}^{(h)}(x_a,Q^2,y|\mathrm{A})=(1-\alpha P_{qT}^2)/(3-\alpha P_{qT}^2). \quad (19)$$

We can see that, as can be expected, if we look only at the 00 component of the spin density matrix, the result is independent of the azimuthal direction of the transverse polarization, i.e., independent of the angle $\phi$. It depends only on the magnitude of the transverse polarization $P_{qT}$ and thus on the variables $x_a$, $Q^2$, and $y$. We see also that $\rho_{00}^{(h)}(\mathrm{A})$ is smaller than 1/3, which is different from the results in the longitudinally polarized case. This shows that, if the fragmenting quark is transversely polarized, the vector meson produced should have a larger probability of being in the helicity $\pm1$ states. As long as there exists a contribution of the vector mesons of group A, the final results for $\rho_{00}^{(h)}(\eta)$ in this frame should also be smaller than 1/3. We know from Fig. 2 that the contributions of group A increase with increasing $\eta$, so we expect a decreasing $\rho_{00}^{(h)}(\eta)$ as $\eta$ increases for vector mesons such as $K^{*+}$ and $\rho^{\pm}$. It is significantly smaller than 1/3 for large $\eta$. Hence, measurements of $\rho_{00}^{(h)}(\eta)$ of vector mesons in transversely polarized $pp$ collisions should give a good check of the extension of the relation of Eq. (1) to the transversely polarized case.

(2) *In the transversity frame*. To compare with the results in the helicity frame for the longitudinally polarized case, we also calculate the spin alignment of the vector mesons in the transversely polarized case in the "transversity frame." In this frame, similar to the helicity frame for the longitudinally polarized case, we take the (transverse) polarization direction of the fragmenting quark as the quantization axis. The spin density matrix of the outgoing quark and that of the antiquark are both diagonal in this frame, and they are given by

$$\rho_q^{(t)}=\frac{1}{2}\begin{pmatrix} 1+P_{qT} & 0 \\ 0 & 1-P_{qT} \end{pmatrix}, \tag{20}$$

$$\rho_{\bar{q}}^{(t)}=\frac{1}{2}\begin{pmatrix} 1-\alpha P_{qT} & 0 \\ 0 & 1+\alpha P_{qT} \end{pmatrix}, \tag{21}$$

where the superscript $(t)$ denotes the transversity frame. We see that they have completely the same form as those for the corresponding quarks and antiquarks in the longitudinally polarized case in the helicity frame. The corresponding spin density matrix $\rho^{(t)}(\mathrm{A})$, which can be obtained from the direct product $\rho_q^{(t)}\otimes\rho_{\bar{q}}^{(t)}$, is also diagonal, and the result for the 00 component $\rho_{00}^{(t)}(\mathrm{A})$ is given by

$$\rho_{00}^{(t)}(x_a,Q^2,y|\mathrm{A})=(1+\alpha P_{qT}^2)/(3-\alpha P_{qT}^2). \tag{22}$$

We note that, since $\rho^{(t)}(\mathrm{A})$ is diagonal in the transversity frame, there is a simple relation between $\rho_{00}^{(h)}(\mathrm{A})$ and $\rho_{00}^{(t)}(\mathrm{A})$. The relation can easily be obtained by using $\rho_{00}^{(h)}(\mathrm{A})=\langle h=0|\rho(\mathrm{A})|h=0\rangle=\Sigma_{m_T,m_T'}\langle h=0|m_T\rangle \times\langle m_T|\rho(\mathrm{A})|m_T'\rangle\langle m_T'|h=0\rangle$ and $\langle m_T|\rho(\mathrm{A})|m_T'\rangle=\rho_{m_T,m_T'}^{(t)} =\rho_{m_T,m_T}^{(t)}\delta_{m_T,m_T'}$. (Here, $h$ denotes the helicity and $m_T,m_T'$ denote the spin components along the quantization axis in the transversity frame.) It is given by

$$2\rho_{00}^{(h)}(x_a,Q^2,y|\mathrm{A})+\rho_{00}^{(t)}(x_a,Q^2,y|\mathrm{A})=1. \tag{23}$$

We see that $\rho_{00}^{(t)}(x_a,Q^2,y|\mathrm{A})$ can also be obtained from this relation and the result for $\rho_{00}^{(h)}(x_a,Q^2,y|\mathrm{A})$ given by Eq. (19). We also see that the $\rho_{00}^{(t)}(x_a,Q^2,y|\mathrm{A})$ given by Eq. (22) is in general larger than 1/3. This means that the probability for the produced vector mesons to be in spin states that are transverse to the moving direction is smaller. We see that, although the values of $\rho_{00}^{(t)}(x_a,Q^2,y|\mathrm{A})$ and $\rho_{00}^{(h)}(x_a,Q^2,y|\mathrm{A})$ are different, they represent the same physical meaning.

We should note that, to measure $\rho_{00}^{(t)}$ in experiments, we need to determine the quantization axis in this frame. Since the quantization axis is chosen as the polarization direction of the quark before fragmentation, it can be different for different vector mesons. The polarization direction is determined by Eq. (14). In practice, we first find the normal direction to the scattering plane and then determine the polarization direction according to the rules described below Eq. (15). (cf. also Fig. 2 of Ref. [23].) The moving direction of the fragmenting quark can be replaced by the thrust axis of the quark jet.

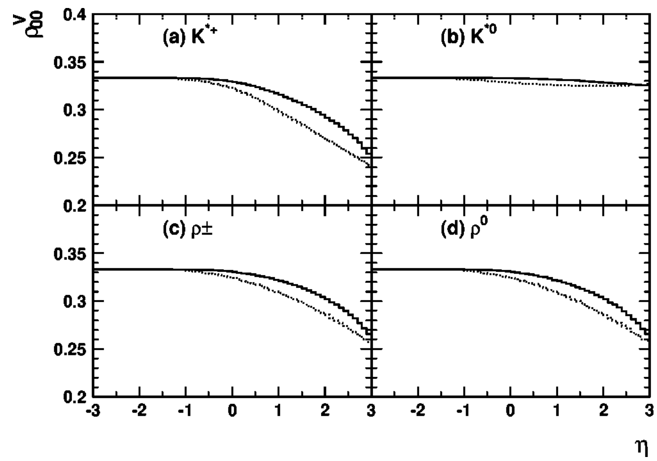


FIG. 4. Spin alignment of different vector mesons with $p_T > 13$ GeV in $pp\to VX$ at $\sqrt{s}=500$ GeV in the helicity frame when one of the proton beams is transversely polarized. The solid lines denote the results obtained using the light-cone quark-spectator model for $\delta q(x)$; the dotted lines correspond to the results obtained using the upper limit in Soffer's inequality.

Inserting Eqs. (19) and (22) into Eq. (12), we can calculate $\rho_{00}(\eta)$ for different vector mesons in the above two frames in $pp$ collisions with one beam transversely polarized. For these calculations, we first need to calculate $P_{qT}$ using Eq. (15). So far, the transversity distribution $\delta q(x,Q^2)$ is not known yet. In order to get a feeling for how large the spin alignment can be, we make an estimation using the simple form for $\delta q(x)$ in the light-cone SU(6) quark-spectator model [24] as input. In this model, $\delta q(x)$ is given by

$$\delta u_v(x)=\left[u_v(x)-\frac{1}{2}d_v(x)\right]\tilde{M}_S(x)-\frac{1}{6}d_v(x)\tilde{M}_V(x),$$

$$\delta d_v(x)=-\frac{1}{3}d_v(x)\tilde{M}_V(x), \tag{24}$$

where $\tilde{M}_S(x)$ and $\tilde{M}_V(x)$ are related to the Melsh-Wigner rotation, which can be calculated according to the procedure given in [25]. The results obtained for $\rho_{00}(\eta)$ for different vector mesons at $\sqrt{s}=500$ GeV and $p_T>13$ GeV in the helicity frame are shown in Fig. 4 and those in the transversity frame are shown in Fig. 5. For comparison, we also show the results obtained by taking the upper limit of $\delta q(x,Q^2)$ in Soffer's inequality [26]

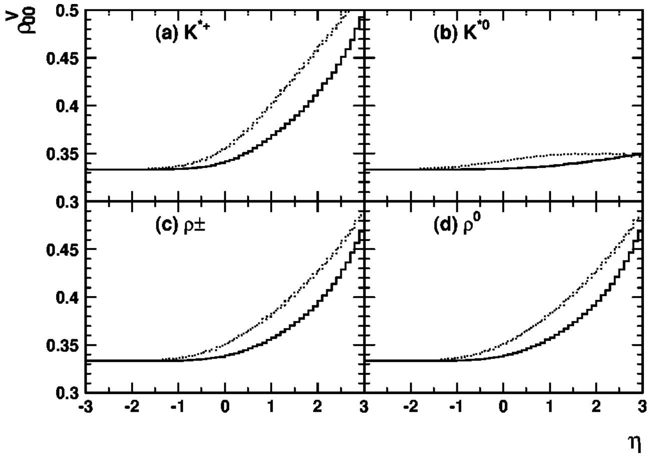


FIG. 5. Spin alignment of different vector mesons with $p_T > 13$ GeV in $pp \to VX$ at $\sqrt{s}=500$ GeV in the transversity frame when one of the proton beams is transversely polarized. The solid lines denote the results obtained using the light-cone quark-spectator model for $\delta q(x)$; the dotted lines correspond to the results obtained using the upper limit in Soffer's inequality.

$$|\delta q(x,Q^2)| \leqslant \frac{1}{2}[\Delta q(x,Q^2)+q(x,Q^2)]. \tag{25}$$

From these figures, we see that, in the helicity frame, the $\rho_{00}$'s of vector mesons are smaller than 1/3 in the large $\eta$ region, but the effect is not very significant. In the transversity frame, however, the $\rho_{00}$'s of $K^{*+}$, $\rho^{\pm}$, and $\rho^0$ are larger than 1/3 and increase to about 0.5 with increasing $\eta$, which is similar to the results for the longitudinally polarized case. However, the results for $K^{*0}$'s $\rho_{00}$ in both frames are much nearer the unpolarized case of 1/3. This is because the $d$ quark fragmentation dominates the $K^{*0}$'s production at high $p_T$, and, according to Soffer's inequality, $|\delta d(x,Q^2)|$ should be small, since $\Delta d(x,Q^2)$ is negative for the valence quark.

As we mentioned above, the gluon is not transversely polarized, so it is much safer to assume that there is no contribution of gluon fragmentation to the spin alignment of vector mesons in transversely polarized $pp$ collisions. Therefore, the high $p_T$ cut that we invoked in the longitudinally polarized case to ensure a small contribution of gluon fragmentation is unnecessary now. Here, only a relative high $p_T$ cut is required to ensure the validity of PQCD. We therefore also make the calculations for $p_T > 8$ GeV at $\sqrt{s}=500$ GeV and the results are given in Fig. 6 and Fig. 7. The results are very similar to those obtained in the case of the higher $p_T$ cut $p_T > 13$ GeV.

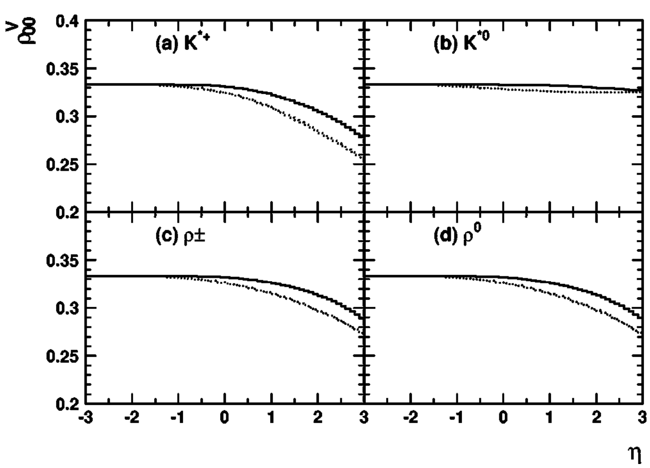


FIG. 6. The same as Fig. 4, but for $p_T > 8$ GeV.

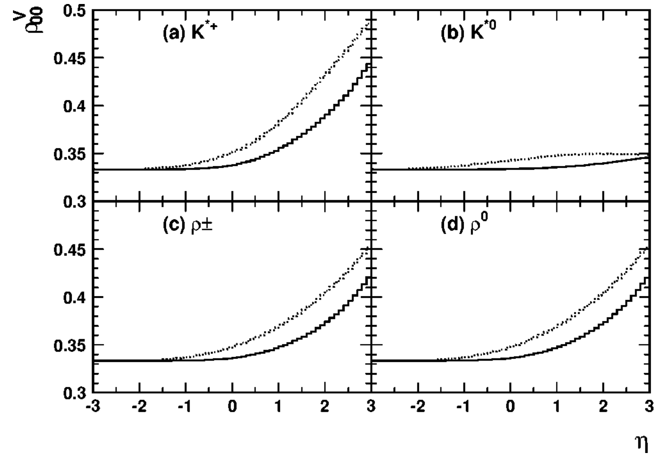


FIG. 7. The same as Fig. 5, but for $p_T > 8$ GeV.

## V. SUMMARY

In summary, we calculated the spin alignment of vector mesons with high $p_T$ in high energy $pp$ collisions with one longitudinally polarized beam by taking the spin of the vector meson as the sum of the spin of the scattered quark and that of the secondary antiquark produced in the fragmentation process. The results for different vector mesons at RHIC energy are presented. These results show that quite significant spin alignment exists for $K^{*+}$, $\rho^{\pm}$, and $\rho^0$. We also studied the spin alignment of vector mesons in the case that one of the proton beams is transversely polarized. Using the same method as that in the longitudinally polarized case, we obtained the results in the helicity and the transversity frames. The results show that, in the large $\eta$ region, the $\rho_{00}$'s in the helicity frame are smaller than 1/3, while the $\rho_{00}$'s in the transversity frame are larger than 1/3. The magnitude of the effect depends on the transversity distribution of the quarks in the proton. It is expected to be more significant in the large $\eta$ region. Its measurement can provide useful information about spin effects in strong interactions and the spin structure of the nucleon.


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